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THREE – DIMENSIONAL FLOW OVER A STRETCHING SURFACE WITH THERMAL RADIATION AND HEAT GENERATION IN THE PRESENCE OF CHEMICAL REACTION AND SUCTION/INJECTION

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ABSTRACT

The effects of thermal radiation, heat generation, chemical reaction and suction/injection on heat and mass transfer characteristics over a stretching surface subjected to three dimensional flow are studied. The governing boundary layer equations are transformed to ordinary differential equations containing thermal radiation parameter, heat generation parameter, suction/injection parameter, stretching ratio parameter, chemical reaction parameter, prandtl and Schmidt number. These equations are solved through applying symbolic computation to Runge-Kutta technique in order to solve four point boundary value problem. The velocity, temperature and concentration profiles are plotted and discussed in details for various values of the different embedded flow parameters.

Keywords: thermal radiation, heat generation, chemical reaction, stretching surface, suction / injection, three dimensional flow.

1. INTRODUCTION

Many important engineering mass transfer processes occur simultaneously with heat transfer. Cooling towers, dryers, and combustors are just few examples of equipment that intimately couple heat and mass transfer. Coupling can arise when temperature-dependent mass transfer processes cause heat to be released or absorbed over a stretching surface. For example during evaporation latent heat is absorbed at a liquid surface when vapor is created. This tends to cool the surface, lowers the vapor pressure, and reduces the evaporation rate. Elbashbeshy et. al. [1-3] studied the effect of internal heat generation and suction/injection on the flow and thermal boundary layer over a stretching surface. Fang et. al. [4] studied the influence of temperature dependent viscosity and thermal conductivity on the boundary layers. Ishak et. al. [5-6] studied the effect of heat flux and suction/injection on the flow and thermal boundary layer over an unsteady stretching surface. Sultana et. al. [7] studied the effect of internal heat generation, suction/injection, and radiation on the flow over a stretching surface embedded in porous medium. Al-Odat et. al. [8] provided a local similarity solution of an exponentially stretching surface with an exponential dependence of the temperature distribution in the presence of the magnetic field effect. Rashed [9] studied the radiative effect on heat transfer from a non-isothermal, arbitrary stretching surface in a porous medium. EL-Arabawy [10] studied the effects of suction/injection and chemical reaction on mass transfer over a stretching surface. All these studies considered the two dimensional flow problem, by developing

the problem to three dimensional flow we found a good lists of references which discussed this problem. Nazar et. al. [11] studied the effects of viscoelastic fluid on the velocity profiles of three dimensional flow over a stretching surface. Takhar et. al. [12] studied the effects of heat transfer on three dimensional MHD boundary layer flow through a stretching surface. Kandasamy et. al. [13] studied the effects of variable viscosity, Heat and Mass transfer on nonlinear mixed convection flow over a porous wedge with heat radiation in the presence of homogenous chemical reaction. Shateyi [14] studied the Thermal Radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. El-dabe et. al. [15] studied the effects of heat generation / absorption and chemical reaction on three dimensional viscoelastic fluid through a stretching surface. Aboeldahab et. al. [16] studied the combined free convective heat and mass transfer effects on the unsteady three dimensional laminar flow over a time dependent stretching surface, also the effect of generation or consumption of the diffusion species due to a homogeneous chemical reaction. Olanrewaju et. al. [17] studied the effects of soret and dufour on an unsteady mixed convection past a porous plate moving through a binary misture of chemically reacting fluid. The purpose of this work is to study the effects of the thermal radiation, heat generation and first order chemical reaction on heat and mass transfer on a steady three dimensional flow over stretching surface. It may be remarked that the present analysis in an extension of and a complement to the earlier papers [7] and [13].

2. FORMULATION OF THE PROBLEM

Consider a steady, laminar, incompressible, and viscous flow on a continuous stretching surface with thermal radiation, heat generation, chemical reaction and suction/injection. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to the other chemical species, the concentration of species far from the surface, C_{∞} is very small [18]. The x- axis, y- axis are run along the plan of a continuous surface, and the z-axis is perpendicular to it as shown in fig (1). The conservation equations for the steady three dimensional flow are



Fig (1) physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$
(1)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = u\frac{\partial^2 u}{\partial z^2}$$
(2)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = u\frac{\partial^2 v}{\partial z^2}$$
(3)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k}{rc_p}\frac{\partial^2 T}{\partial z^2} - \frac{1}{rc_p}\frac{\partial q_r}{\partial z}$$

$$+\frac{Q}{rc_p}(T-T_{\infty}) \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - k_1(C - C_\infty)$$
(5)

Subjected to the boundary conditions

$$u = U_{w}, \quad v = V_{w}, \quad T = T_{w}, \quad C = C_{w}, \quad at \quad z = 0$$
$$u = 0, \quad v = 0, \quad \frac{\partial u}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad T = T_{\infty},$$
$$C = C_{\infty}, \quad as \quad z \to \infty$$
(6)

Where u, v and w are velocity components in the x, y, and z directions, respectively, v is the viscosity, T is the temperature of the fluid, k is the thermal conductivity, ρ is the density of fluid, c_p is the specific heat due to constant pressure, Q is the heat source or sink, C is the concentration

of the flow, D is the mass diffusion coefficient, and k_1 is the reaction rate coefficient.

It is assumed that the viscous dissipation is neglected, and the physical properties of the fluid are constant. Also we considered here the case of thick optically medium with optical thickness $t = a \ z \implies 1$, therefore the Rosseland approximation for radiation radiative heat flux is simplified as

$$q_r = -\frac{4s}{3a} \frac{\partial T^4}{\partial z} \tag{7}$$

Where σ and α are the Stefen-Boltzman constant and the mean absorbation coefficient respectively.

We assume that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_{∞} and neglecting higher order terms we get

$$T^{4} \cong 4TT_{m}^{3} - 3T_{m}^{4} \tag{8}$$

Using equation (7) and (8) in the energy eguation (4) becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k}{rc_p} \frac{\partial^2 T}{\partial z^2} + \frac{16 s T_{\infty}^3}{3 a r c_p} \frac{\partial^2 T}{\partial z^2} + \frac{Q}{rc_p} (T - T_{\infty})$$
⁽⁹⁾

We assume that the stretching velocities U_w , and V_w , the flow Temperature T_w , and Concentration C_w are of the form

$$U_{w} = ax, \qquad V_{w} = by, \quad T_{w} = T_{\infty} + c_{1}x^{b} = T_{\infty} + c_{2}y^{b}.$$

$$C_{w} = C_{\infty} + d_{1}x^{g} = C_{\infty} + d_{2}y^{g}$$
(10)

Where a and b are constant and called stretching rate, c_1 , c_2 , d_1 , and d_2 are constant and β , γ are the temperature and concentration parameters.

We now introduce the following dimensionless functions f, g, φ and θ , and the similarity variable η

$$h = \sqrt{\frac{a}{u}} z, \quad u = (ax) f'_{(h)}, \quad v = (ay) g'_{(h)},$$

$$w = -\sqrt{au} (f + g), \quad j_{(h)} = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \quad q_{(h)} = \frac{C - C_{\infty}}{C_{w} - C_{\infty}} \quad (11)$$

Where prime denotes the differntiation with respect to η , using (11) the mass conservation equation (1) is indentically satisfied, and substituting into eqs. (2,3,5,and 9) we obtain

$$f''' + (f+g)f'' - f'^2 = 0$$
⁽¹²⁾

$$g''' + (f+g)g'' - g'^2 = 0 \tag{13}$$

$$\left(\frac{4R}{3}+1\right)j'' + \Pr[(f+g)j' - b(f'+g')j + (dj)] = 0$$
(14)

$$q'' + Sc[(f+g)q' - g(f'+g')q] - Lq = 0$$
⁽¹⁵⁾

where $(\mathbf{R} = k \alpha/4\sigma T_{\infty}^{3})$ is the thermal radiation parameter, $(\delta = \mathbf{Q} / a \mu c_{p})$ is the dimensionless heat source or sink, $(\mathbf{Pr} = \mu c_{p} / k)$ is the prandtl number $(\mathbf{Sc} = \upsilon/\mathbf{D})$ is the schmidt number, $(\mathbf{L} = k_{1} \mathbf{Sc} / a)$ is th chemical reaction parameter.

The boundary condition (6) become

$$f(0) + g(0) = I, \quad f'(0) = 1, \quad g'(0) = z, \quad j(0) = 1, \quad q(0) = 1$$

$$f'(\infty) = 0, \quad f''(\infty) = 0, \quad g'(\infty) = 0, \quad g''(\infty) = 0, \quad j(\infty) = 0,$$

$$q(\infty) = 0$$
(16)

Where λ is the suction/injection parameter, ($\lambda > 0$) corresponds to suction, and ($\lambda < 0$) corresponds to injection

Where $(\zeta = b/a)$ is the stretching ratio parameter, and when $(\zeta = 0)$ the problem reduces to the two-dimensional case [g = 0], and when $(\zeta = 1)$ the problem reduces to the axisymmetric flow [f = g]

3. NUMERICAL SOLUTIONS AND RESULTS

We first convert the Equations (9-12) into a system of linear equations of first order, by using

$$\begin{split} s_1 &= f, \ s_2 = f', \ s_3 = f'', \ s_4 = g, \ s_5 = g', s_6 = g'', s_7 = j \\ s_8 &= j', \ s_9 = q, \ s_{10} = q' \end{split}$$

$$\begin{split} S_{1}' &= S_{2} \\ S_{2}' &= S_{3} \\ S_{3}' &= S_{2}^{2} - (S_{1} + S_{4})S_{3} \\ S_{4}' &= S_{5} \\ S_{5}' &= S_{6} \\ S_{6}' &= S_{5}^{2} - (S_{1} + S_{4})S_{6} \\ S_{7}' &= S_{8} \\ S_{8}' &= \left(\frac{3P_{r}}{4R + 3}\right) \left[b\left(S_{2} + S_{5}\right)S_{7} - (S_{1} + S_{4})S_{8} + -dS_{7} \right] \\ S_{9}' &= S_{10} \\ S_{10}' &= Sc \left[g\left(S_{2} + S_{5}\right)S_{9} - (S_{1} + S_{4})S_{10} - LS_{9} \right] \end{split}$$
(17)

Subjected to The initial Conditions

$$S_{1}(0) + S_{4}(0) = I, \quad S_{2}(0) = 1, \quad S_{3}(0) = m, \quad S_{5}(0) = z,$$

$$S_{6}(0) = n, \quad S_{7}(0) = 1, \\ S_{8}(0) = h, \\ S_{9}(0) = 1, \quad S_{10}(0) = k$$
(18)

where m, n, h and k are unknown to be determined as a part of the numerical solution

Using **MATHEMATICA**, we define a function F[m_, n_, h_, k_]:= NDSolve [system17&18]. The value of m, n, h and k are determined upon solving the equations, S2(η max) =0, S5(η max) =0, S7(η max) =0 and S9(η max) =0. Once m, n, h and k are determined, the system is closed. It can be solved numerically using the ND Solve. Consequently , only one integration pass is enough to solve the problem instead of an iteration technique like shooting method .

The computations have been carried out for a various values of the stretching parameter (ζ), the suction parameter (λ), the thermal radiation parameter (R), the dimensionless heat source or sink (δ), and the chemical reaction parameter (L).

To validate the numerical method used in this study, the case of ($\lambda = 0, R= 0, \delta=0$, and L=0) was considered in table (1) and the results for *-f* "(0) and *-g*"(0) are compared with the numerical solution which reported in Nazar [11].

<u>Table (1)</u>: Values of $-f''_{(0)}$, $-g''_{(0)}$ for a various values of

		(5)	
	ζ	Nazar [11]	present results
	0.00	1.0013	1.0000
- f ''(0)	0.50	1.0935	1.0931
	1.00	1.1738	1.1737
	0.00	0.0000	0.0000
- g "(0)	0.50	0.4656	0.4652
	1.00	1.1738	1.1737

Also the concentration surface gradient values for a various values of concentration parameter (γ) for suction/injection case was considered in table (2) and the results are compared with the exact solution which reported in Al-Arabawy [8].

<u>Table (2)</u>: Values of $-\theta'(0)$ for a various values of (α) at Sc=0.7, $\zeta =0$, L =0.8

λ	γ	L	Al-Arabawy [8]	present results
	-2	0.00	1.04672	1.04672
1	-1	0.20	1.25353	1.25353
1	1	0.80	1.62421	1.62421
	2	1.20	1.79220	1.79220
	-2	0.00	0.06266	0.06266
1	-1	0.20	0.42367	0.42367
-1	1	0.80	0.95568	0.95568
	2	1.20	1.16929	1.16929

From the engineering point of view, the most important characteristics of the flow are the skin friction coefficient, Nusselt and Sherwood numbers which are indicate physically to surface shear stress, rate of heat and mass transfer respectively.

3.1 Surface Shear Stress
$$t_{w(x)} = m \left(\frac{\partial u}{\partial z}\right)_{z=0} = \frac{r U_w^2}{\sqrt{R_e}} f_{(0)}^{"}$$

and $t_{w(y)} = m \left(\frac{\partial v}{\partial z} \right)_{z=0} = \frac{y r U_w^2}{x \sqrt{R_e}} g_{(0)}^{"}$

since the skin friction coefficient is given by $C_f = \frac{2t_w}{rU_w^2}$

i.e
$$f''(0) = \frac{1}{2}C_f \sqrt{R_e}$$
 and $g''(0) = \frac{1}{2}\frac{x}{y}C_f \sqrt{R_e}$

3.2 Surface Heat Flux

$$q_{w} = -k \left(\frac{\partial T}{\partial z}\right)_{z=0} = -\frac{k(T_{w} - T_{w})}{x} \sqrt{R_{e}} \quad j'_{(0)}$$

since the Nusselt number is given by $Nu = \frac{1}{k}$

$$u = \frac{xq_w}{k(T_w - T_w)}$$

i.e
$$-j'(0) = \frac{Nu}{\sqrt{R_e}}$$

3.3 Surface Mass Flux

$$M_{w} = -D\left(\frac{\partial C}{\partial z}\right)_{z=0} = -\frac{D(C_{w} - C_{\infty})}{x}\sqrt{R_{e}} \quad q_{(0)}$$

since the Sherwood number is given by Sh = -

$$Sh = \frac{xM_w}{D(C_w - C_w)}$$

i.e
$$-q'(0) = \frac{Sh}{\sqrt{R_e}}$$

Table (3) : Values of - $\phi'(0)$ for a various values of (\delta) at Pr=0.71, ζ =0.5 , R =1, and β = γ =2

λ	δ	- φ'(0)
	-0.50	1.06516
15	-0.20	0.98616
1.5	0.20	0.84075
	0.50	0.63784
	-0.50	0.79623
15	-0.20	0.72432
-1.5	0.20	0.68197
	0.50	0.67378

 $\begin{array}{l} \underline{Table~(4)}: Values~of~-~\phi'(0)~~for~a~various~values~of~~(R)~~at\\ Pr=0.71,~\zeta=0.5~,~\delta=0.5,~and~\beta=\gamma=2 \end{array}$

λ	R	- φ'(0)
	0.30	1.20762
1.5	0.50	0.96960
	0.70	0.70110
	0.30	0.75023
-1.5	0.50	0.67916
	0.70	0.62410

<u>Table (5)</u>: Values of $-f''_{(0)}$, $-g''_{(0)}$, $-\phi'_{(0)}$, $-\theta'_{(0)}$ for a various values of (ζ) at Pr=0.71, Sc=0.7 δ =0.5. R =1, L= 0.8, β=γ=2

		0 - 0.5, 1	(-1, L-0.0)	p_{-1}^{-2}	
λ	ζ	- f ''(0)	- g ''(0)	- φ'(0)	- θ'(0)
15	0	2.00000	0.00000	0.27788	1.90509
1.5	0.5	2.08966	0.99111	0.43244	2.09819
15	0	0.50000	0.00000	0.40380	0.96774
-1.5	0.5	0.54765	0.18185	0.52575	1.20082



Fig. (1): The velocity profile with increasing of stretching ratio parameter (ζ).



Fig. (2): the temperature profile with increasing of stretching ratio parameter (ζ).



Fig. (3): the Concentration profile with increasing of stretching ratio parameter (ζ).



Fig. (4): The velocity profile with increasing of suction/injection parameter (λ).



Fig. (5): the Concentration profiles with increasing of suction/injection parameter (λ).



Fig. (6): The temperature profile with increasing of the thermal radiation parameter (R).



Fig. (7): The temperature profile with increasing of heat source or sink parameter (δ).



Fig. (8): The temperature profile with increasing of the prandlt number (Pr).



Fig. (9): The Concentration profiles with increasing of the chemical reaction parameter (L).



Fig (10): The Concentration profiles with increasing of the Schmidt number (Sc).

4. DISCUSSIONS

The influence of the suction/injection parameter (λ) , stretching parameter (ζ) , the thermal radiation parameter (R), the dimensionless heat source or sink (δ), the prandlt number (Pr), chemical reaction parameter (L), and the schmidt number (Sc) on the dimensionless velocity, the dimensionless temperature and the dimensionless concentration are shown in figures (1-10) for temperature parameter (β =2) and concentration parameter (γ =2).

Figures (1-3) show the effect of the stretching parameter (ζ), on the velocity, temperature and concentration. We observe that the increase of the stretching parameter decreases velocity, temperature, and concentration. Also it is clear that the velocity, temperature and concentration in the case of two dimensional problem (ζ =0) are high compared to the three dimensional problem (ζ =0).

Figures (4-5) show the effect of suction/injection parameter (λ) on the velocity and the concentration respectively. We observe that the increase of the suction/injection parameter increases the velocity and decrease the concentration. Also it is clear that the velocity in the case of suction is higher than that in the case of injection while the concentration in the case of injection is higher than that in the case of suction.

Figures (6-7) show the effect of thermal radiation parameter (R) and the dimensionless heat source or sink (δ), on the temperature respectively. We observe that the increases of the thermal radiation parameter produces an increase in the thermal condition of the flow and its thermal boundary layer. Also increasing of heat source or sink parameter increases the temperature. Also it is clear that the temperature in the case of heat source is higher than in the case of sink.

Figure (8) shows the effect the prandlt number (Pr) on the temperature. We observe that the increase of prandlt number decreases the temperature. Which mean that the type of flow has a direct effect on the boundary layer temperature.

Figure (9) shows the effect chemical reaction parameter (L) on the concentration. We observe that the increase of the chemical reaction parameter decreases the concentration.

Figure (10) shows the effect of different chemical species on the concentration profiles, where Diffusing chemical species of most interest in air has Schmidt number in the range from 0.1 to 10. We observe that the concentration decreases with the increasing of Schmidt number. Also it is clear that the concentration boundary layer decreases with increase of Schmidt number.

Table (5) shows that the values of $-f''_{(0)}$ and $-g''_{(0)}$ increases with the increase of suction/injection parameter (λ) and stretching parameter (ζ), therefore the skin friction coefficient decreases with increase of suction/injection parameter and stretching parameter, which leads to decrease of surface shear stress. Also it is clear that the values of $-f''_{(0)}$, $-g''_{(0)}$ for 3-D problem are higher than that of 2-D

problem, which mean that the skin friction coefficient and surface shear stress are high in 3-D problem compared to 2-D problem.

Tables (3) and (4) show that the values of temperature gradient at the surface $-\phi'_{(0)}$ increases with the decrease of thermal radiation parameter (R), and the dimensionless heat source or sink (δ) and increases with increase of suction/injection parameter (λ), therefore the Nuseslt number decreases with increase thermal radiation parameter (R), and the dimensionless heat source or sink and decrease of suction/injection parameter, which leads to increase of surface heat flux with decrease of thermal radiation parameter, and the dimensionless heat source or sink and increase of suction/injection parameter. Also it is clear that the values of $-\phi'_{(0)}$ for 3-D problem are higher than that of 2-D problem, which mean that Nuseslt number and surface heat flux are high in 3-D problem compared to 2-D problem. Also The concentration gradient at the surface $-\theta'(0)$ shown in table (2) decreases with the increase of suction/injection parameter (λ), chemical reaction parameter (L) and concentration parameter (γ), therefore the Sherwood number decreases with increase of suction/injection parameter, chemical reaction parameter, and concentration parameter. which leads to decrease of surface mass flux. Also it is clear that the values of $-\theta'_{(0)}$ for 3-D problem are higher than that of 2-D problem, which mean that Sherwood number and surface mass flux are high in 3-D problem compared to 2-D problem.

5. CONCLUSION

Numerical solution has been obtained for the effects of the thermal radiation , heat generation, suction/injection, and chemical reaction on mass and heat transfer characteristics over a stretching surface. It has been found that these parameters hav a considerable effect on surface shear stress, rate of heat transfer, and rate of mass transfer. The temperature, concentration, and velocity are also affected by these parameters.

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8. NOMENCLATURE

Т	fluid temperature (K)
С	fluid concentration $(k \text{ mol.m}^{-3})$
k	thermal conductivity $\left(W \ m^{-1} K^{-1} \right)$
C_p	specific heat $(m^2 s^{-2} K^{-1})$
Q	volumetric rate of heat generation
D	mass Diffusion coefficient (m^2s^{-1})
K_1	chemical reaction coefficien t (sec^{-1})
f	dimensionless function [-]
g	dimensionless function [-]
R	thermal radiation parameter $\left(\frac{ka}{4sT_{\infty}^3}\right)$ [-]
L	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-]
L Greek	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols
L Greek υ	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right) (m^2/s^{-1})$
L Greek υ μ	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)$ (m ² /s ⁻¹) dynamic viscosity (N.Sec/m ²)
L Greek υ μ	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)$ (m ² /s ⁻¹) dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³)
L Greek υ μ ρ η	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)$ (m ² /s ⁻¹) dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³) similarity variable [-]
L Greek υ μ ρ η Ψ	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right) (m^2/s^{-1})$ dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³) similarity variable [-] stream function [-]
L Greek υ μ ρ η ψ j	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)$ (m ² /s ⁻¹) dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³) similarity variable [-] stream function [-] dimensionless function [-]
L Greek ν μ ρ η ψ j q	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)$ (m ² /s ⁻¹) dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³) similarity variable [-] stream function [-] dimensionless function [-]
L Greek ν μ ρ η ψ j q s	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)\left(m^2/s^{-1}\right)$ dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³) similarity variable [-] stream function [-] dimensionless function [-] stefen boltzman constant [-]
L Greek ν μ ρ η ψ j q s α	chemical reaction parameter $\left(\frac{k_1Sc}{a}\right)$ [-] symbols kinematic viscosity $\left(\frac{\mu}{\rho}\right)\left(m^2/s^{-1}\right)$ dynamic viscosity (N.Sec/m ²) density of fluid (kg/m ³) similarity variable [-] stream function [-] dimensionless function [-] stefen boltzman constant [-] absorbation coefficient [-]

gtemperature parameter [-]bconcentration parameter [-]dheat source parameter
$$\left(\frac{Q}{amc_p}\right)$$
 [-]1suction/injection parameter $-\left(\frac{w}{\sqrt{au}}\right)$ [-]zstretching parameter $\left(\frac{b}{a}\right)$ [-]

subscripts

- *w* surface conditions
- ∞ condition far away from the surface

Superscript

1

differentiation with respect to η

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